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ABSTRACT

SIMULTANEOUS FORECASTING OF PRICE MAGNITUDE AND DIRECTION USING TWO-STAGE OLS-PROBIT

The ability to accurately forecast price hinges largely on an ability to reflect both dimensions of price, magnitude and direction, in a forecasting model. This paper demonstrates how price magnitude and direction can be captured in a simultaneous forecasting system. Empirical results reflect the importance of the simultaneous forecasting procedure.
The ability to anticipate the future is an agelong problem which pervades all areas of life, especially economic well-being. In most commercial endeavors, including agricultural pursuits, this largely focuses on our ability to anticipate future prices.

For this reason there have been advances in the field of price forecasting in recent years. Three primary price forecasting approaches include causal models, noncausal models such as the auto-regressive-integrated moving-average model (ARIMA), and composite models. Oliveira et al. employed the ARIMA technique to forecast beef prices, while the forecasting ability of large scale econometric models were compared to the forecasting ability of the futures market by Just and Rausser and also by Martin and Garcia. Bessler and Brandt forecast livestock prices using a model that combined an ARIMA model with a causal model. Other researchers [Naylor, et al.; Bechtner and Rutner] have compared Box-Jenkins models with causal models.

These cited works focused on accuracy of forecasting price magnitude. However, Tomek and Robinson have pointed out that price has two dimensions, the first is magnitude and the second is direction. It follows that the ability to forecast accurately hinges largely on an ability to reflect both dimensions of price in a forecasting approach.

A recent advance was made in this direction by Menkhaus and Adams who used discriminant analysis to predict turning points and included these predictions in a price forecasting equation to enhance the accuracy of the price forecast. The approach depicted in this paper captures the idea
conveyed by Menkhaus and Adams in a more theoretically appealing simultaneous conceptualization.

The purpose of this paper is to show how price magnitude and direction can be captured in a simultaneous forecasting system. The paper is organized as follows: The specification of the simultaneous forecasting of price magnitude and direction is couched according to Heckman's simultaneous equations system encompassing a dichotomous endogenous variable. This is followed by an explanation of the two-stage econometric procedure specified by Maddala for estimating the model. An empirical example, concerning forecasting in September the price of March yearlings and the probability that prices will move up or down between October and March, is used to demonstrate the simultaneous forecasting procedure. The empirical example is followed by a discussion of data requirements, estimation of the model, and an evaluation of the predictive power of the model. The final section involves conclusions regarding the usefulness of the simultaneous price magnitude and direction forecasting model.

The Model

The methodology for this paper follows Heckman's two-stage simultaneous estimation procedure and can best be described using a formal model:

(1) \[ Y_1 = \gamma_1 Y_2^* + \beta_1' X_1 + u_1 \]
(2) \[ Y_2^* = \beta_2' X_2 + u_2 \]

where

- \( Y_1 \) = price magnitude in time t+1;
- \( Y_2^* \) = a latent variable observed only in dichotomous form such that
  \[ Y_2^* = 1 \text{ if } Y_2 > 0 \]
  \[ Y_2^* = 0 \text{ otherwise; } \]
- \( X_1 \) = vector of explanatory variables for \( Y_1 \);
\[ X_2 = \text{vector of explanatory variables for } Y_2; \]
\[ \gamma_1 = \text{scalar coefficient}; \]
\[ \beta_1, \beta_2 = \text{vectors of coefficients}; \]
\[ u_1, u_2 = \text{random error terms with bivariate normal distribution}. \]

The reduced form is written:

(3) \[ Y_1 = \pi_1 X + v_1 \]
(4) \[ Y_2^* = \pi_2 X + v_2 \]

where \( X = \text{combined exogenous variables in } X_1 \text{ and } X_2; \)

\[ \pi_1, \pi_2 = \text{vector of reduced form coefficients}; \]

\[ v_1, v_2 = \text{random error terms with bivariate normal distribution}. \]

Identification of this model requires that \( u_1 \) and \( u_2 \) be independent, or else there is at least one variable in \( X_2 \) not included in \( X_1 \) (Maddala).

Equation (2) forecasts the direction of price movement. This predicted directional variable is endogenous to the price magnitude equation (1).

Estimation

Heckman's two-stage estimation process provides consistent estimates for equations (1) and (2). It should be noted that in this particular model the reduced form (equation 4) and the structural form (equation 2) are the same, thus the estimator \( \hat{\beta}_2 \) is identical to the estimator \( \hat{\beta}_2 \). The estimation process begins by obtaining an estimate of \( \beta_2 \). However, since \( Y_2^* \) is observed only as a dichotomous variable, we can only estimate \( \pi_2 / \sigma_2 \) and \( \beta_2 / \sigma_2 \), where \( \sigma_2^2 = \text{var}(v_2) \). Hence, equation (2) is estimated by probit Maximum Likelihood (ML) in the form,

\[ Y_2^{**} = \frac{Y_2^*}{\sigma_2} = \frac{\beta_2}{\sigma_2} X_2 + \frac{u_2}{\sigma_2}. \]

Equation (1) is now written as

(6) \[ Y_1 = \gamma_1 \sigma_2 Y_2^{**} + \beta_1' X_1 + u_1. \]
In the second-step of the estimation process, equation (1) is estimated by OLS after the predicted index value of $Y_{2}^{**}, \frac{\hat{\beta}_2}{\sigma_2} X_2$, is substituted into the equation. The estimable parameters in this model are $\gamma_1 \sigma_2, \beta_1, \frac{\beta_2}{\sigma_2}, \sigma_1$, and $\frac{\sigma_{12}}{\sigma_2}$.

The asymptotic covariance matrix for equation (5) is that provided by probit ML estimation of the equation. However, the asymptotic covariance matrix for equation (6) must be derived using a procedure similar to the one used by Amemiya for a simultaneous tobit model.

The covariance matrix of the two-stage estimates of equation (6) can be estimated by,

\begin{equation}
\text{Var}(\hat{\alpha}_1) = c(H'X'XH)^{-1} + (\gamma_1 \sigma_2)^2 (H'X'X)^{-1} H'X'X \sigma_1 \sigma_2 (H'X'XH)^{-1}
\end{equation}

where

- $\alpha_1' = (\gamma_1 \sigma_2, \beta_1')$;
- $H = (\beta_2, J_1)$;
- $J_1 = \text{matrix consisting of 1's and 0's so that } XJ_1 = X_1$;
- $c = \sigma_1^2 - 2 \gamma_1 \sigma_1 \sigma_{12}$;
- $\sigma_{12} = \text{covariance matrix of the probit ML estimate of } \beta_2$ and $\sigma_2$

\begin{equation}
\text{Cov}(\alpha_1, \alpha_2) = \begin{bmatrix}
\sigma_1^2 & \sigma_{12} \\
\sigma_{12} & \sigma_2^2
\end{bmatrix}
\end{equation}

Application of Model

The model postulated above appears to be a reasonable framework with which to address the specific problem of whether feeder calf producers should sell spring calves in the fall or over-winter calves for the following spring sale.
Important economic variables for this application are based on previous research [Kearl; Menkhaus and Adams]. The two-equation simultaneous system is specified as follows:

(8) \[ \text{MARCHYL}_{t+1} = f(\text{DIFF}_{t+1}, \text{SEPTSL}_t, \text{SEPTC}_t, \text{INV}_t, \text{SLTR}_t) \]

(9) \[ \text{DIFF}_{t+1} = f(\text{MSDIR}_t, \text{SEPTSL}_t, \text{SEPTC}_t, \text{INV}_t, \text{SLTR}_t) \]

where

\[ \text{MARCHYL}_{t+1} = \text{deflated price of March yearlings in year } t+1 \text{ (cwt.);} \]

\[ \text{SEPTSL}_t = \text{deflated price of September slaughter steers in year } t \text{ (cwt.);} \]

\[ \text{SEPTC}_t = \text{deflated price of September corn in year } t \text{ (cents per bu.);} \]

\[ \text{INV}_t = \text{January 1 cattle and calf inventory in year } t \text{ (1,000 head);} \]

\[ \text{SLTR}_t = \text{cattle and calf commercial slaughter in year } t \text{ (1,000 head);} \]

\[ \text{DIFF}_{t+1} = \text{dichotomous variable representing the difference in the March}_{t+1} \text{ yearling price and October}_t \text{ yearling price (1 indicates a price rise between October}_t \text{ and March}_{t+1} \text{ and 0 signals a price decrease); in equation (8) estimation, DIFF is } \beta_2 \frac{x_2}{\sigma_2}, \text{ an index;} \]

\[ \text{MSDIR}_t = \text{Dichotomous variable representing direction of movement in yearling prices between March and September in year } t \text{ (1 if up and 0 if down).} \]

The price of feeder steers (MARCHYL_{t+1}) is a function of the endogenous variable (DIFF_{t+1}) indicating the direction of feeder steer price movement between October_t and March_{t+1}, the price of slaughter cattle (SEPTSL_t) and the input costs of feeding the cattle to slaughter.
weight which can be represented by the cost of corn ($SEPTC_t$).
Additionally, the inventory ($INVT_t$) and slaughter ($SLTR_t$) variables are used to depict expansion and liquidation phases in cattle inventories. The price direction endogenous variable ($DIFF_{t+1}$) is a function of the prices of slaughter steers and corn, cyclical liquidation indicators, and the direction of price movement ($MSDIR_t$) between March$_t$ and September$_t$.

In our application of the model, the conditions for identification are met since equation (8) has one less exogenous variable than equation (9).

Data

The model was estimated over the period 1926 to 1981. The years 1982 through 1985 were reserved for post sample prediction testing. Cattle statistics were obtained from selected issues of the U.S.D.A. publications Livestock and Meat Statistics and Livestock and Meat Situation. Corn prices were obtained from the Commodity Year Book. Consumer price index data are from the Department of Commerce Survey of Current Business.

The variable ($MARCHYL$) is the deflated March yearling price (cwt.) at Kansas City in year $t+1$. To calculate the observed values of the dichotomous variable ($DIFF$) the weighted average of all weights and grades of March feeder steer prices (deflated) at Kansas City in year $t+1$ and the October price (deflated) of good and choice feeder steer calves at Kansas City in year $t$ were compared. The movement in yearling prices ($MSDIR$) was calculated using deflated Kansas City March and September yearling prices in year $t$. The variable ($SEPTC$) is the September 15 average deflated price for corn received by U.S. farmers. The deflated September slaughter price ($SEPTSL$) represents the average cost per 100 lbs. of sales out of first hands for choice slaughter steers at Chicago for 1926 through 1949. Over the period 1950 to 1985 this price represents the price of choice slaughter
steers at Omaha, 900-1100 lbs. The inventory (INV) variable is the January cattle and calf inventory (1,000 head). Slaughter (SLTR) represents the sum of cattle and calf commercial slaughter (1,000 head).

Results

The estimation results of the application model using the two-stage Probit-OLS estimation method are presented in Table 1. As expected, estimation of the simultaneous system resulted in a highly significant coefficient for the predicted endogenous directional variable (DIFF) in equation (8). Additionally, in equation (8) the sign of SEPTSL was expected to be positive. That is, if the price of slaughter steers increase, then the price of feeder calves should move in the same direction. The sign of SEPTC was hypothesized to be negative since as the price of corn increases the price of feeder steers should decline. Inventory was expected to be positive while the slaughter variable was expected to be negative. The dependent variable in equation (9) is price direction which is the second component of price. Consequently, the coefficient signs are the opposite of those in equation (8) and reflect a tempering of the composite effect resulting in the price forecast.

The primary test, however, of any forecasting model is its predictive power. The forecasting ability of the model is evaluated according to how well price magnitude and direction were predicted for the in-sample period of 1926 to 1981 and the post sample period of 1982 to 1985.

Different classification criteria may be used to evaluate the directional equation (9). A 50-50 criterion means that if the predicted probability of price direction is 0.50 or greater and the actual value for the dichotomous variable is 1, the price direction prediction is correct. Given a 50-50 criterion, the probit ML equation properly classified 18 of
Table 1. Estimation Results for the Price Magnitude and Direction Model, 1926-1981

<table>
<thead>
<tr>
<th>Variable</th>
<th>Price Magnitude Equation (8)</th>
<th>Probability of Price Direction Equation (9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-5.6169 (-0.9087)$^a$</td>
<td>1.5146 (0.9730)</td>
</tr>
<tr>
<td>DIFF</td>
<td>2.8939 (6616.3100)</td>
<td></td>
</tr>
<tr>
<td>MSDIR</td>
<td></td>
<td>0.9094 (1.9330)</td>
</tr>
<tr>
<td>SEPTSL</td>
<td>1.0864 (0.1675)</td>
<td>-0.1011 (-2.2080)</td>
</tr>
<tr>
<td>SEPTC</td>
<td>-0.0543 (-0.2619)</td>
<td>0.0125 (1.9860)</td>
</tr>
<tr>
<td>INV</td>
<td>0.2522E-3 (0.0082)</td>
<td>-0.9576E-4 (-2.6270)</td>
</tr>
<tr>
<td>SLTR</td>
<td>-0.4228E-3 (-2.4390)</td>
<td>0.2345E-3 (2.6160)</td>
</tr>
</tbody>
</table>

$^a$Numbers in parenthesis are the asymptotic t statistics.

$^b$F test and Likelihood Ratio test were significant at $\alpha = 0.05$. 

$R^2 = 0.67$  \hspace{1cm} Log-Likelihood$^b = -26.108$

$F^b = 20.19$

$U_2 = 0.15$
the 24 upward price movements for the in-sample period 1926-1981, and correctly classified 28 of the 32 downward price movements making a combined 82 percent correct directional classification. Additionally, the model correctly predicted 17 of the 25 turning points.

The evaluation of the predictive power of the directional equation for the in-sample period is delineated in Table 2. The percentage of accurate directional predictions remains relatively high even when the classification criterion becomes restrictive. Even the most restrictive criterion of 90-10 yields a 68 percent accuracy rate.

The price magnitude equation is evaluated using Theil's $U_2$ statistic. For the in-sample period 1925-1980, the price magnitude equation had a Theil's $U_2$ coefficient of (0.15) which indicated that the price forecasting equation is considerably better than a naive no-change model in predicting price magnitude (a value of 1 represents equivalency with a naive no-change extrapolation model).

The in-sample results of the model appear to be encouraging with respect to the simultaneously forecasting of the direction of price movement between October feeder steer prices in time $t$ and March yearling prices in time $t+1$ and the forecasting of March yearling prices. However, the most crucial test of our model is whether it can predict in the post sample period, in our case 1982-1985. All available observations were used in the forecasting procedure. Data through 1981 were used to estimate the model to forecast for 1982. Data through 1982 were used to forecast price magnitude and direction for 1983, and so on. Chow tests indicated that the parameter estimates were stable. The validation results for this period are in Table 3. The directional equation correctly classified all 4 of the cases. The price magnitude equation yielded a Theil's $U_2$ of (0.09).
Table 2. Evaluation of Directional Probability Prediction, 1926-1981

<table>
<thead>
<tr>
<th>Classification Criterion&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Percentage of Accurate Directional Predictions</th>
</tr>
</thead>
<tbody>
<tr>
<td>50-50</td>
<td>82</td>
</tr>
<tr>
<td>60-40</td>
<td>75</td>
</tr>
<tr>
<td>70-30</td>
<td>73</td>
</tr>
<tr>
<td>80-20</td>
<td>71</td>
</tr>
<tr>
<td>90-10</td>
<td>68</td>
</tr>
</tbody>
</table>

<sup>a</sup>For example, the 60-40 criterion means that the predicted probability was correct 75 percent of the time during the 1926-81 period. That is, the predicted probability was 0.60 or greater when the actual March yearling price in year t+1 was greater than or equal to the actual October yearling price in year t and 0.40 or less when the actual March yearling price in year t+1 was less than the actual October yearling price in year t.
Table 3. Predictions of Price Magnitude and Direction, 1982-1985

<table>
<thead>
<tr>
<th>Year</th>
<th>Price Magnitude</th>
<th>Price Direction Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual</td>
<td>Predicted</td>
</tr>
<tr>
<td></td>
<td>(1967 dollars per cwt.)</td>
<td></td>
</tr>
<tr>
<td>1982</td>
<td>22.12</td>
<td>25.31</td>
</tr>
<tr>
<td>1983</td>
<td>22.36</td>
<td>20.57</td>
</tr>
<tr>
<td>1984</td>
<td>20.58</td>
<td>20.08</td>
</tr>
<tr>
<td>1985</td>
<td>21.10</td>
<td>20.59</td>
</tr>
</tbody>
</table>

\[ u_2 = 0.09 \]

\[ ^a \text{All post sample direction predictions were correct.} \]
Concluding Remarks

The purpose of this paper was to demonstrate a method that allows the simultaneous prediction of price magnitude and direction in order to enhance the usefulness of forecasting results. The idea of forecasting price direction and incorporating it into a price magnitude forecast is not only theoretically correct and intuitively appealing but also provides the producer with additional information. Often times, information concerning the direction of price movement is just as important to the decisionmaker as the actual price itself. Predicted price direction and magnitude in tandem may even promote the expansion of firm marketing alternatives and, in our particular example, help producers contend with the cyclical nature of cattle production.
References


